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EVALUATION OF METHODS FOR PREDICTING DEVIATION ANGLE IN PLANAR TURBINE CASCADES

This paper presents a review of methods for predicting the flow exit angle from a planar turbine cascade operating with subsonic and transonic flow regimes. An analysis of empirical correlations used to determine the deviation angle is performed, and the advantages and limitations of each approach are discussed. The accuracy of empirical correlations and the CFD approach is evaluated by comparing their predictions with experimental data obtained for the VKI turbine cascade profile under both design and off-design operating conditions. The comparison reveals a low prediction accuracy of empirical correlations for considered regimes. The largest discrepancies are observed at outlet Mach numbers below 0.6. The closest agreement with experimental data is achieved using CFD (AxCFD™). The results show good consistency both in terms of the absolute values of the deviation angle and the trend of its variation under off-design conditions. Additional CFD simulations of three planar turbine cascade configurations demonstrate a significant influence of blade geometry on the predicted deviation angle. The relationship between the kinetic energy loss coefficient and the deviation angle is also discussed. Both CFD and experimental results indicate that the minimum loss condition corresponds to the minimum deviation angle. Furthermore, a consistent trend in the variation of these two parameters is observed across different operating regimes. It is concluded that accurate prediction of the deviation angle is a complex problem requiring consideration of a wide range of geometric and operating parameters of the cascade. Further research in this area is recommended.

Key words: axial turbine, turbine cascade, deviation angle, outlet angle, CFD, AxCFD™, validation.

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АНАЛІЗ МЕТОДІВ ВИЗНАЧЕННЯ КУТУ ВІДХИЛЕННЯ
ДЛЯ ПЛОСКИХ РЕШІТОК ТУРБІН

У статті виконано огляд методів визначення кута виходу робочого тіла з плоскої турбінної решітки для дозвукових та трансзвукових режимів течії. Проведено аналіз емпіричних залежностей для визначення кута відхилення, відзначено переваги та недоліки кожної методики. Оцінено точність емпіричних кореляцій та CFD-підходу шляхом зіставлення результатів, отриманих із використанням цих методів, з експериментальними даними для профілю турбінної решітки VKI в розрахункових і позарозрахункових режимах. У результаті порівняння встановлено низьку точність прогнозування використаних емпіричних кореляцій як для розрахункових, так і для позарозрахункових режимів роботи решітки профелю VKI. Найбільші похибки спостерігаються при числах Маха на виході менше 0,6. Найкраща узгодженість з експериментальними даними отримана для результатів CFD (AxCFD™). Результати добре збігаються як за абсолютними значеннями кута відхилення, так і за характером його зміни в позарозрахункових режимах. Додаткові CFD-розрахунки трьох конфігурацій плоских турбінних решіток показують значний вплив геометрії профілю на визначення кута відхилення. У роботі також розглянуто зв'язок між коефіцієнтом втрат кінетичної енергії та кутом відхилення. Результати CFD та експериментальні характеристики профілів свідчать про збіг режиму роботи, при якому спостерігається мінімум втрат і мінімум кута девіації. Також відзначається узгодженість трендів зміни цих параметрів у різних режимах роботи решітки. Зроблено висновок, що визначення кута відхилення є складною задачею, яка потребує врахування широкого спектра геометричних і режимних параметрів решітки. Рекомендується продовження досліджень у цьому напрямку.

Ключові слова: осьова турбіна, решітка турбіни, кут відхилення, кут виходу, CFD, AxCFD™, валідація.

Nomenclature

b – blade chord;
 b_c – blade chord (by camber);
 s – cascade pitch;
 o – throat opening;
 t_{max} – profile max thickness;
 x_c – max thickness location from leading edge;
 β – flow angle;
 β_g – gauging angle;
 β_m – metal angle angle;
 γ – stagger angle;
 δ_g – deviation angle measured from β_g ;
 δ_m – deviation angle measured from β_{2m} ;
 θ – blade camber;
 e – radius of curvature of the profile on the suction side downstream throat;
 l – blade height;
 t – temperature;

p – pressure;
 ρ – density;
 ν – kinematic viscosity;
 w – outlet velocity;
 M – Mach number;
 Re – Reynolds number (wb/ν);
 ζ – loss of kinetic energy;
 μ – flow coefficient;
 $AVDR$ – axial velocity density ratio (out.-to-in.);
 index “1” – inlet;
 index “2” – outlet;
 index “is” – isentropic expansion.

Introduction

The flow exit angle is one of the key parameters of turbine airfoil cascades. An error in estimating this parameter may lead to a deviation of the actual turbine performance from the predicted values.



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The outlet flow angle β_2 in turbomachine cascades is determined using the deviation angle δ , which, depending on the methodology, is measured relative either to the gauging angle β_g or to the blade exit metal angle β_{2m} :

$$\beta_2 = \beta_g + \delta_g, \quad (1)$$

$$\beta_g = \arcsin\left(\frac{o}{s}\right), \quad (2)$$

$$\beta_2 = \beta_{2m} + \delta_m. \quad (3)$$

As a rule, formulas (1) and (2) define angles measured relative to the tangential direction, whereas formula (3) refers to angles measured relative to the axial direction.

An analysis of the literature shows that the exit flow angle depends on the downstream distance from the cascade and is governed by a specific combination of geometric parameters and flow operating regime.

The estimations of the deviation angle reported in the literature show a significant spread in its absolute value.

According to Aungier [1], the deviation angle δ_g is quite low for the turbine cascades with $\beta_g \sim 10^\circ - 20^\circ$ but can reach $5^\circ - 6^\circ$ at gauging angles close to 50° .

A synthesis of experimental and computational (CFD) studies conducted by Islam-Sjolander [2] for cascades with outlet metal angles in the range of $14^\circ - 41^\circ$ indicates a more pronounced effect. The deviation angles δ_m are in the range of $1.6^\circ - 5.5^\circ$.

When comparing information from different sources, it is also necessary to consider a certain degree of difference in the flow pattern between planar cascades of turbine profiles and cylindrical cascades with complex three-dimensional blade geometry. In particular, a specific control of the flow distribution within the cascade passage can lead to a significant difference between the gauging angle and the actual exit flow angle (Fig. 2) [3].

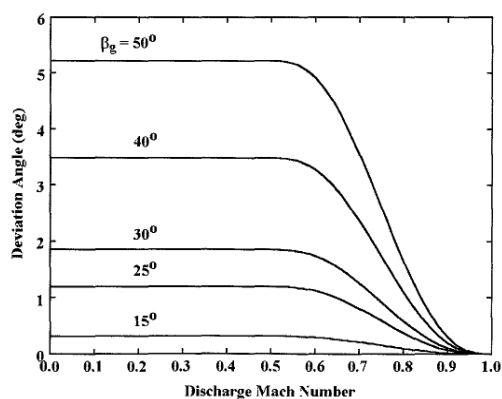


Fig. 1 – Deviation angle (δ_g) depending on the discharge Mach number for the different gauging angles [1]

In practice, various correlations are used to estimate the deviation angle, most of which have been derived from the processing of experimental data for planar cascades. The applicability of these correlations is limited to the corresponding cascade types and the parameter ranges for which they were obtained.

With the development of computational fluid dynamics (CFD) methods, it has become possible to analyze the flow structure in cascades in much greater detail and to obtain quantitative estimates of parameters with accuracy comparable to that of physical experiments. However, CFD simulations are typically applied to known cascade geometries, which must first be designed by the engineer. Therefore, despite the capabilities of CFD, empirical correlations remain relevant due to their high computational efficiency and convenience for use at the steps of preliminary design and flow path optimization.

The aim of the work

The aim of this work is to review existing methods and approaches for determining the flow exit angle in planar turbine cascades and to assess the accuracy of their predictions for subsonic and transonic flow regimes.

The results of data analysis and CFD simulations can be used to refine engineering methods, thereby improving the accuracy of calculations in turbine stage design.

The general part

1 Review of empirical correlations for the exit flow angle prediction

There is a large number of empirical correlations in the literature for predicting the flow angle at the outlet of the turbine cascade.

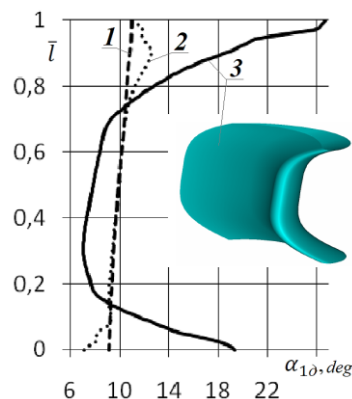


Fig. 2 – Flow angle distribution along the airfoil span [3]: 1 – gauging angle; 2 – exit flow angle (prismatic blade); 3 – exit flow angle (blade with complex 3D lean)

One of the most frequently cited work on the methodology for determining energy losses and the exit flow angle is by Ainley-Mathieson [4].

According to the Ainley-Mathieson method, the main factors determining the exit angle are the exit Mach number, the ratio of o/s , or indirectly the gauging angle, as well as the profile curvature e on the suction side downstream the throat.

The results in the original work are presented in the form of graphs and analytical dependencies.

Wilson [5] transformed the Ainley-Mathieson data for practical use in the following form:

$$- M_2 < 0.5$$

$$\alpha_{(0 < M < 0.5)} = \frac{7}{6} \left\{ \cos^{-1} \left(\frac{o}{s} \right) - 10^\circ \right\} + 4^\circ \left(\frac{s}{e} \right); \quad (4)$$

$$- M_2 = 1$$

$$\alpha_{(M=1)} = \left| \cos^{-1} \left(\frac{o}{s} \right) - \left(\frac{s}{e} \right)^{1.786+4.128 \left(\frac{s}{e} \right)} \right| \sin^{-1} \left(\frac{o}{s} \right); \quad (5)$$

$$- 0.5 < M_2 < 1$$

$$\alpha_2 = \alpha_{(0 < M < 0.5)} - (2M_2 - 1) \left(\left| \alpha_{(0 < M < 0.5)} \right| - \alpha_{(M=1)} \right). \quad (6)$$

The model is applicable for turbine cascades but does not consider a number of geometric and operational regimes for the profile (Reynolds number, incidence angle, etc.).

The main difficulty in using this correlation is the need to determine the curvature radius e . This is challenging at the preliminary design step or during performance analysis of the existing design, where the profile is often defined by a set of coordinate points, or when the profile has a straight-line suction-side segment in the region downstream the throat.

To address this issue, Aungier [1] proposed a modification of the model. The presented formulation introduces some simplifications that allow predicting the deviation angle without detailed profile geometry:

$$- M_2 < 0.5$$

$$\delta_g = \delta_{g,0}; \quad (7)$$

$$- 0.5 < M_2 < 1$$

$$\delta_g = \delta_{g,0} \left[1 - 10X^3 + 15X^4 - 6X^5 \right], \quad (8)$$

where

$$\delta_{g,0} = \arcsin \left\{ \left(\frac{o}{s} \right) \left[1 + \left(1 - \frac{o}{s} \right) \left(\frac{\beta_g}{90} \right)^2 \right] \right\} - \beta_g, \quad (9)$$

$$X = 2M_2 - 1. \quad (10)$$

The level of the deviation angle for this two-parametric model for different gauging angles and Mach numbers is shown in Fig. 1. It can be noted that the deviation angle is small for gauging angles below 25° , approximately 1° . It is also worth noting the tendency for the deviation angle to decrease in the range of $M_2 = 0.5-1.0$, which is the most commonly used for converging cascades.

A key feature of this model is the zero deviation angle at the operating condition of $M_2 = 1$ for all blade profiles with all gauging angles β_g . Aungier is explained this by the assumption that for the $M_2 = 1$ regime the energy losses in the region downstream the throat section are insignificant and the change in density can be neglected. However, there is no practical confirmation of such behavior of the deviation angle at Mach numbers close to 1.0.

Equations (7) – (10) are quite convenient to use, particularly at the preliminary design step. However, they do not account for many geometric and operating flow regimes of the cascade.

The approach by Howell and Carter-Huges [6], [7] was mainly developed based on data for compressors, however, some authors point out the possibility of using dependencies for the deviation angle for turbine cascades [7]. The developed empirical dependence, including the deviation angle in the so-called «design point mode» δ_m , arising at the design incidence angle:

$$\delta_m = m\theta \left(\frac{t}{b_c} \right)^n, \quad (11)$$

where n – is a coefficient, equal to 0.5 for compressor stators and equal to 1 for inlet guide vanes (can also be applied for turbine cascades [7]);

The coefficient m depends on the geometric parameters of the blade cascade and is calculated as follows

$$m = 0.23 \left(2 \frac{x_c}{b_c} \right)^n + \frac{\beta_2}{500}. \quad (12)$$

The exit angle in this equation is defined relative to the outlet metal angle of the cascade. Angles in this study are measured from the axial direction.

It is worth noting the direction of works performed in Carleton University that are related to the use of the AVDR parameter as a base [2], [8]. This parameter determines the degree of flow expansion in the cascade and is defined as the ratio of the axial velocity multiplied by the density at the outlet to a similar complex at the inlet.

Based on Rodger's cascade experiments, it was observed a linear dependence on AVDR at the design incidence angle. This relationship can be described by the following equation [2]

$$\delta_m = \delta_0 + 14(\text{AVDR} - 1). \quad (13)$$

When the incidence angle varies, the observed dependence can be represented by the following cubic-type equation [2]

$$\delta_m = \delta_0 (\text{AVDR})^3. \quad (14)$$

It should be noted that the deviation angle in Fig. 3 and Fig. 4 is negative, since in Rodger's experiments it was defined as $\beta_2 - \beta_{2m}$. In [2], these data are presented with the opposite (positive) sign, for which equations (13) and (14) were derived.

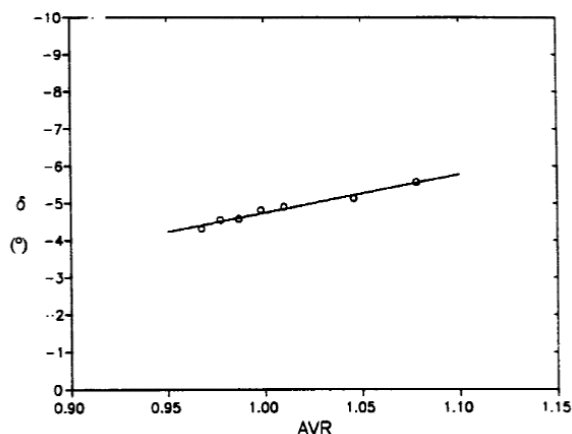


Fig. 3 – The effect of AVDR on midspan exit flow deviation at design incidence [8]

Taking into account results obtained by Rodger, Islam-Sjolander proposed a correlation that considers AVDR as well as the influence of a number of cascade's geometric parameters:

$$\delta_m = \frac{(AVDR)^3 \left(\frac{o}{s}\right)^{1.1} (\beta_1 + \beta_{2m})^{2.25}}{\gamma^{1.45} \left(\frac{t_{\max}}{b}\right)^{0.3} (22 + 0.22\beta_{1m}^{1.64})}. \quad (15)$$

Angles in this formula are measured from the exit metal angle.

It should be noted that dependence (15) was obtained by the authors based on a relatively limited amount of data. A total of 45 cases of both experimental studies of cascades and the results of numerical flow modeling (RANS) were considered.

As discussed in [2], [9], the correlation shows acceptable agreement for AVDR 0.95 – 1.05, when $M_2 < 0.7$.

The model (15) was further improved by Zhu-Sjolander [9]:

$$\delta_m = 17.3 \frac{\left(\frac{o}{s}\right)^{0.05} (\beta_1 + \beta_{2m})^{0.63} \cos^2(\gamma) \left(\frac{t_{\max}}{b}\right)^{0.29}}{\left(30 + 0.01\beta_{1m}^{2.07}\right) \tanh\left(\frac{Re_2}{200000}\right)^2}. \quad (16)$$

The authors note that the new correlation incorporates a larger dataset (81 cases), which allows for more accurate prediction of the effects of low Reynolds numbers, high blade loading, and extend the range of validity of the correlation in general. The authors omitted the AVDR parameter due to its minimal variation (0.95 – 1.05) for the cascades used in the process.

The variation of geometric parameters used to obtain the correlation (16) is presented in [9]. The range of blade stagger angles can be noted as $62.1^\circ - 18.8^\circ$. From this point of view, applying equation (16) may result in significant deviations for active profile

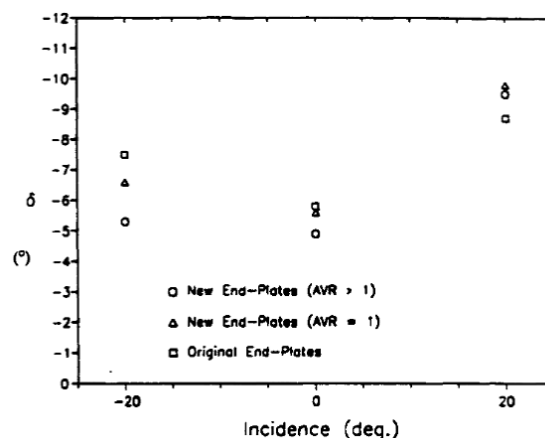


Fig. 4 – Variation of the midspan deviation with incidence [8]

cascades what have blade profile stagger angle below $< 18.0^\circ$.

When comparing the results of formula (16) with the data of 81 profiles, the authors point out a deviation in the results at positive incidence angles [9].

Reference [10] recommends a correlation formulated using an analytical approach. By applying the mass conservation equations with a number of assumptions, the exit angle for subsonic flow regimes is determined using the following formula:

$$\alpha_2 = \arcsin\left(\frac{\mu}{\sqrt{1-\zeta}} \frac{\rho_{2, is}}{\rho_2} \sin \beta_g\right). \quad (17)$$

There are a lot of computational models [4], [7], [10], [11], for determining the kinetic energy loss coefficient ζ , which differ in the level of geometric and operating factors detalization. Calculating the cascade flow coefficient μ is more challenging. There are significantly fewer methods available for its determination compared to the number of models for energy loss and deviation angle.

As an example, the methodology in [10] can be described. The flow coefficient of the turbine blade cascade depending on the cascade type can be determined by the following formulas:

– stator cascade:

$$\mu = 0.982 - 0.0056 \frac{b}{l_2} - \Delta\mu_M - \Delta\mu_{Re}; \quad (18)$$

– rotor cascade:

$$\mu = 0.965 - 0.01 \frac{b}{l_2} - \Delta\mu_M - \Delta\mu_{Re} - \Delta\mu_{\Delta\beta}. \quad (19)$$

Exit Mach number factor:

$$\Delta\mu_M = 0.01M_{2, is}^2 - 0.005M_{2, is}^3. \quad (20)$$

Exit Reynolds number factor:

$$\Delta\mu_{Re} = -8.0Re_2^{-0.5}. \quad (21)$$

Profile camber angle factor (rotor only):

$$\Delta\mu_{\Delta\beta} = -0.02 + 0.027 \sin(\Delta\beta), \quad (22)$$

where the channel bend angle $\Delta\beta$:

$$\Delta\beta = 180^\circ - (\beta_1^{opt} - \beta_g), \quad 90^\circ \leq \Delta\beta \leq 150^\circ. \quad (23)$$

The channel bend angle is calculated using the optimal inlet flow angle, which, as reported in [10], exceeds the inlet metal by $3^\circ - 6^\circ$ (the measurement system relates to the tangential direction)

$$\beta_1^{opt} = \beta_{1m} + (3-6)^\circ. \quad (24)$$

Alongside empirical methods, CFD approaches are widely used. RANS methods enable computationally efficient simulations and provide acceptable agreement with experimental data [12] – [14]. Despite this, CFD currently remains a tool for validating the performance of 3D geometries and for supplementing or correcting the empirical correlations used in 1D/2D aerodynamic analysis of turbomachinery flow paths.

This review demonstrates the diversity of methods and tools available to engineers. Of practical interest is the assessment of the accuracy of results obtained using the presented empirical correlations, as well as CFD calculation systems through comparison with experimental data.

The authors carried out such work and made a comparison for the data of experiments, summarized in the work [15].

The AxCFD™ module of the commercial AxSTREAM® software package [16] was selected as the CFD tool. The correctness of the obtained results was confirmed by performing validation, the results of which are presented below.

2 AxCFD™ validation

CFD numerical modeling tools are widely used in modern approaches to the design of high-performance turbomachines. One of such tools is the AxCFD™ software module of the commercial software suite SoftInWay Inc. AxSTREAM® [16].

AxCFD™ enables automatic discretization of the computational domain using a structured hexahedral mesh. The mathematical framework of the program is based on solving the RANS equations for simulating the flow of a real viscous fluid, applying a two-parameter turbulence model. Despite the fact that AxCFD™ has been developed and used for over 20 years, there is limited public information regarding its validation cases. In this work, the authors performed such an assessment for the turbine cascade operating in different flow regimes.

AxCFD™ validation was performed using the results of experimental studies conducted on VKI (VKI-Sieverding airfoil) profile. Different geometrical variants of this profile have been tested by many scientific organizations in experimental studies of the turbomachine cascades aerodynamics carried out in wind tunnels (GO – Goettingen, BS – Braunschweig, OX – Oxford, RG – Rhode-St. Genese). As a result of

these studies, extensive information is available on the characteristics of the profile, its geometry, with a detailed description of the test conditions. This information is summarized in [15].

During this work, the testing conditions of the GO university were simulated.

To define the boundary conditions for the CFD simulation, data from the graphs in paper [15], including ζ , β_g , M_1 , M_2 , Re_2 , were digitized. Using the known total temperature at the inlet t_1^* , the inlet total pressure p_1^* and the static pressure p_2 at the outlet of the computational domain were determined based on ideal gas relations and by the following equations:

$$t_2^* = t_1^*, \quad (25)$$

$$t_2 = \frac{t_2^*}{1 + \frac{k-1}{2} M_2^2}, \quad (26)$$

$$w_2 = M_2 \sqrt{k R t_2}, \quad (27)$$

$$\mu_2 = \mu_{ref} \left(\frac{t_2}{t_{ref}} \right)^{\frac{3}{2}} \frac{t_{ref} + C}{t_2 + C}, \quad (28)$$

$$\rho_2 = \frac{Re_2 \mu_2}{w_2 b}, \quad (29)$$

$$p_2 = \rho_2 R t_2, \quad (30)$$

$$p_1^* = p_2 \left[1 - \frac{\left(1 + \frac{k-1}{2} M_2^2 \right)^{-1}}{1 - \zeta} \right]^{\frac{k}{k-1}}. \quad (31)$$

To simulate the working fluid properties, the ideal gas model (air) was used. The $k-\omega$ SST turbulence model was selected. The computational domain was a flat cross-section with one cell along the height (Fig. 5). The boundary layer thickness was selected to ensure the parameter $Y^+ < 1$ in all simulated regimes.

During the study, the influence of mesh size on the prediction of the exit flow angle from the cascade (Fig. 6) and the kinetic energy losses (Fig. 7) was investigated. The analysis showed that grid independence for these two monitored parameters is achieved at a mesh size of approximately 110,000 cells.

When comparing the results, a close match was obtained for both the exit angle (Fig. 8) and kinetic energy losses (Fig. 9). The comparison was carried out with subsonic and transonic flow regimes at the cascade exit.

The conducted computational study showed that the AxCFD™ software provides acceptable accuracy for quantitative assessment as well as for capturing the qualitative flow pattern.

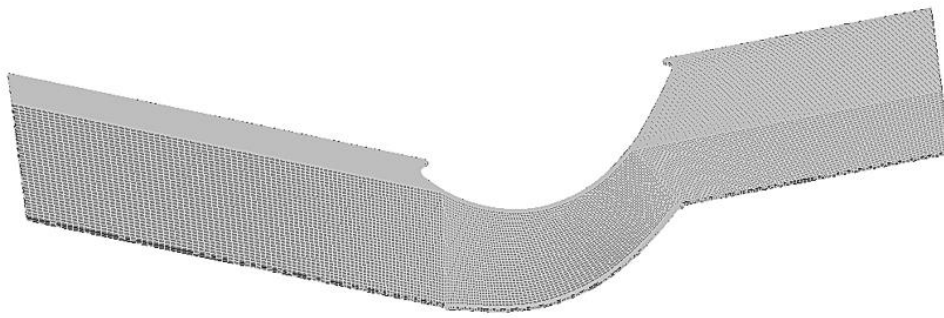


Fig. 5 – Computational mesh

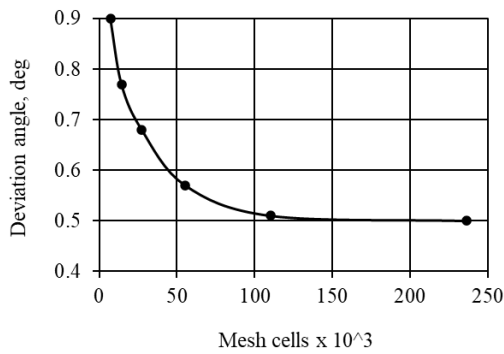


Fig. 6 – Deviation angle depending on the mesh cells number (GO, $M_2 = 0.958$)

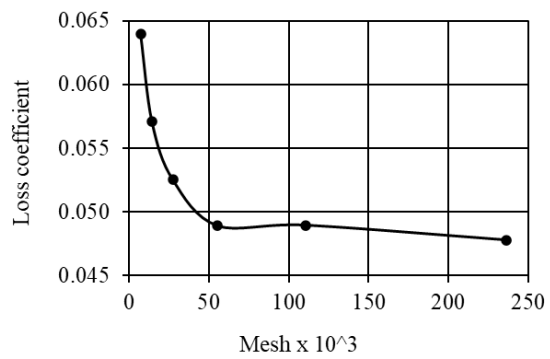


Fig. 7 – Loss coefficient depending on the mesh cells number (GO, $M_2 = 0.958$)

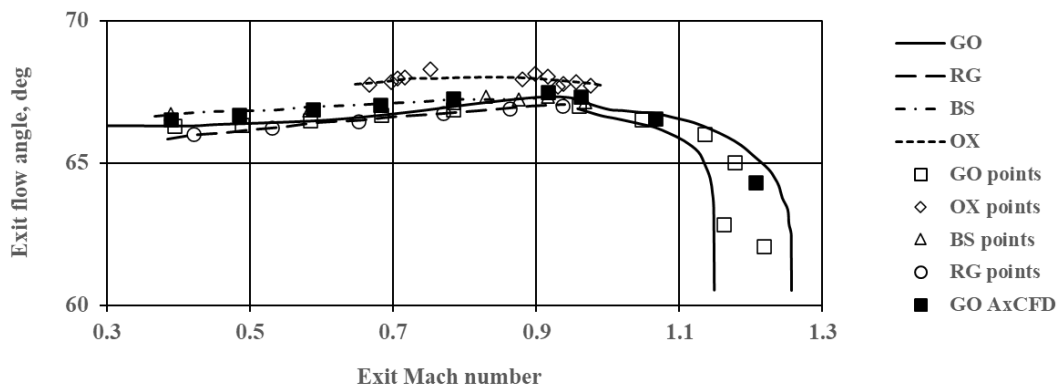


Fig. 8 – Exit flow angle (axial) depending on the exit Mach number

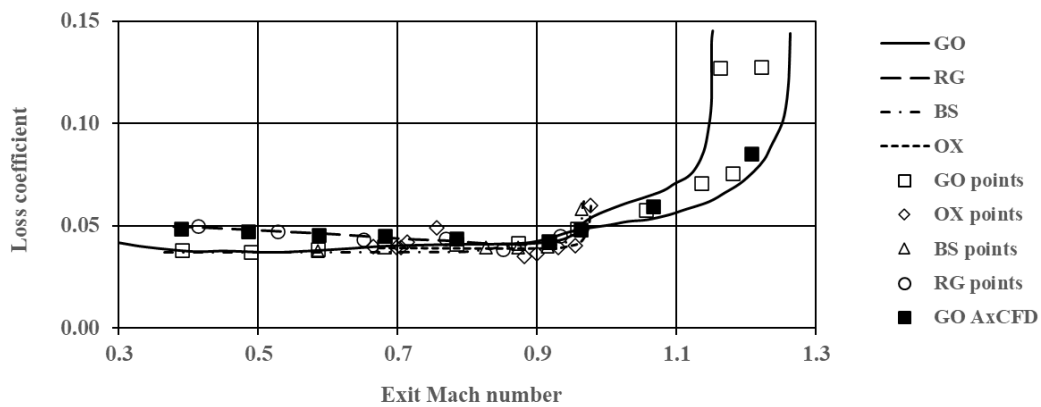


Fig. 9 – Loss coefficient depending on the exit Mach number

3 Evaluation of methods for predicting deviation angle in planar turbine cascades

The comparison of the results obtained from the given empirical correlations and CFD or the blade profile data [15] was performed in two steps.

At the first step, a comparison was carried out for the VKI profile operating mode close to the design point. These results are described in detail in [15]. The results are presented for each research organization.

It should be noted that the data for the VKI profile cascades presented in Table 1 exhibit some differences in certain geometric parameters and, accordingly, in the Reynolds number. The accuracy of the experimental exit angle measurements is in the range of $0.06^\circ - 0.22^\circ$.

Table 2 shows a comparison of the regimes close to the design mode. All data were calculated in accordance with the original methodology described above and then converted into the deviation angle referenced to the gauging angle (1) and to the coordinate system in which angles are measured from the tangential direction.

The closest match with the experiment was obtained for Islam-Sjolander and the AxCFDTM results.

The highest deviation in results was obtained for Carter-Hughes model. The deviation angle is overestimated by a substantial amount.

At the second step, a comparison of the methods was carried out in terms of their ability to predict the behavior of the deviation angle under off-design operating conditions, in this case with respect to the exit Mach number (GO data).

The Carter-Hughes model was not considered at the second phase, as its formulation does not allow

prediction of the deviation angle under off-design conditions.

The approach used to calculate the boundary conditions for these regimes is shown in Section 2 of this article (formulas (25) – (31)).

The results of comparison are shown in Fig. 10. According to the results, the closest match for deviation angle behavior was obtained for CFD and Aungier.

In the Islam-Sjolander model, a slight tendency for the deviation angle to increase with increasing Mach number is also observed due to variations in the AVDR parameter. It should be noted that the variation of this parameter in the tests is small (0.92 – 0.98 for GO [15]), and the cubic exponent applied to this parameter in equation (16) does not allow for capturing such a level of deviation angle variation considering wide range of Mach number.

The Zhu-Sjolander model is also relatively insensitive to changes in operating conditions. This is due to the lack of Mach number effects in the formulation and the relatively small variation of the Reynolds number in the tests ($4 \cdot 10^5 - 8 \cdot 10^5$ for GO).

The Shcheglyayev model shows low deviation and only minor changes in the predicted results when the cascade operating conditions vary. The loss coefficient ζ for formula (17) was calculated in accordance with the recommendations described in [10] (for the rotor cascade, type B).

In general, all empirical models exhibit significant deviations in predicting the deviation angle in the region of Mach numbers below 0.6.

Table 1 – Accuracy of parameters measured in experiments [15]

Parameter	RG	GO	BS	OX
Chord, mm	32.6	60	100	100
Outlet flow angle in ax. (tan.) direction, deg	$67.03 (22.97) \pm 0.15$	$67.02 (22.98) \pm 0.22$	$67.33 (22.67) \pm 0.06$	$67.76 (22.24) \pm 0.1$
Gauging angle in ax. (tan.) direction, deg	$67.74 (22.27) \pm 0.10$	$67.92 (22.08) \pm 0.11$	$67.96 (22.04) \pm 0.03$	$67.96 (22.04) \pm 0.03$
Pitch, mm	23.15 ± 0.04	42.58 ± 0.19	77.88 ± 0.10	77.88 ± 0.10
Stagger angle, deg	33.14 ± 0.09	33.56 ± 0.12	33.29 ± 0.03	33.29 ± 0.03
Loss coefficient	0.046 ± 0.007	0.049 ± 0.001	0.039 ± 0.003	0.040 ± 0.003
M_2	0.933 ± 0.006	0.957 ± 0.003	0.828 ± 0.003	0.932 ± 0.004

* tan. – flow angle measured from the tangential direction; ax. – flow angles measured from the axial direction.

Table 2 – Deviation angle δ_g for the regime close to the design point

Parameter	Reference	Equation	RG	GO	BS	OX
Experim. data	[15]	–	0.71	0.9	0.63	0.20
AxCFD	–	–	0.51	0.51	0.67	0.58
Aungier	[1]	(7) – (10)	0.02	0.01	0.20	0.02
Carter-Huges	[6]	(11) – (12)	8.18	8.63	8.25	8.25
Islam-Sjolander	[2]	(15)	1.04	1.00	0.61	0.40
Zhu-Sjolander	[9]	(16)	3.01	2.94	2.80	2.80
Shcheglyayev	[10]	(17) – (24)	0.09	0.13	0.01	0.06

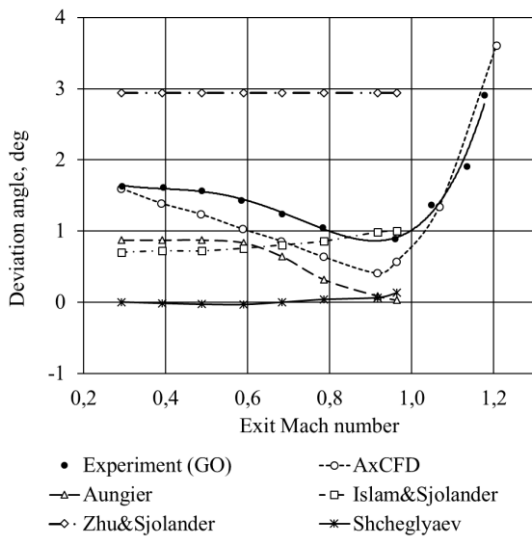


Fig. 10 – Deviation angle results comparison

Results discussion

Based on the figures and table in Section 2, the closest agreement is achieved using CFD methods, both in terms of the absolute value of the deviation angle and the trend of its variation with the exit Mach number.

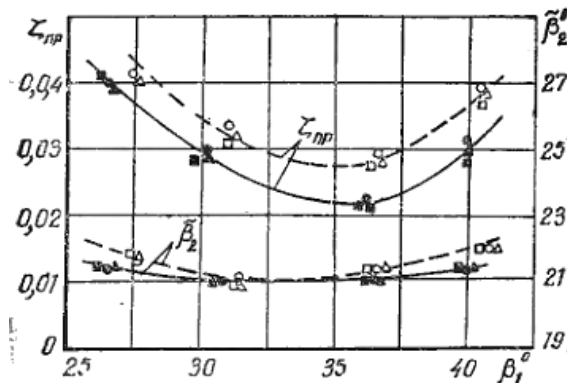


Fig. 11 – Exit flow angle β_2 and kinetic energy losses ζ_{np} depending on the inlet flow angle for IMMK cascade [17]:
 - - - - 1I, — 1MMK

To verify the relationship between the considered parameters, additional CFD simulations of three planar turbine cascades were performed: (Table 3): H2 [18], P2 [3], [18], [19] and a rotor blade profile P9025B (designed initially to operate in transonic regime, suitable for the 25° outlet flow angle (tan.) and 90° (tan.) of inlet flow angle).

The blade profiles were scaled and positioned to ensure identical chord b_c , pitch s and throat opening o .

It should be noted that the Aungier model correctly predicts the trend of deviation angle variation with M_2 . In particular, it captures the weak variation to $M_2 < 0.5$ and the minimum of deviation angle closer to $M_2 = 1$ region. In this regard, a promising direction is the further refinement of the model under baseline operating conditions:

- improving the prediction of the deviation angle at $M_2 = 0.5$ with consideration of geometric and operating parameters of the turbine cascade;
- refining the minimum deviation angle value for the cascade and identifying the corresponding regime (M_2 , Re_2 , incidence angle) at which this minimum occurs.

Analysis of the CFD results and experimental data for the VKI profile reveals a clear correlation between the trends of the kinetic energy loss coefficient and the deviation angle (Fig. 8 and Fig. 9). The local minimum and overall behavior of the deviation angle closely correspond to those of the losses.

A similar relationship can be observed for the data presented in [17]. For the 1I profile and its modification 1MMK – the local minimum is located around the minimum of profile loss (Fig. 11).

A similar trend is also observed in the graphs of parameter distribution along the airfoil span. The variation of the exit angle follows the same tendency as the losses (Fig. 12) [3].

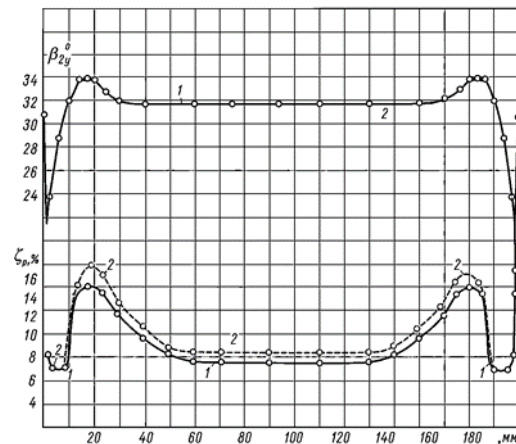


Fig. 12 – Exit flow angle β_{2y} and kinetic energy losses ζ_p distribution along the airfoil span [3]

The objective of this comparison was to eliminate the influence β_g (o/s), which is commonly treated as a key governing parameter in empirical correlations, and to assess the extent to which the profile geometry itself affects the deviation angle.

The inlet angle in the computational model was set to ensure zero incidence, to avoid flow separation and reduce the influence of unsteady effects on the simulation results.

The flow simulations were carried out under the boundary conditions and operating regimes described in Section 2.

Fig. 13 and Fig. 14 present the results for the deviation angle δ_g and kinetic energy losses. It can be seen that the location of the minimum loss and minimum deviation angle is preserved for each profile within the subsonic regime.

The trends of these parameters are generally consistent with those predicted by Aungier's correlations. However, the values of the minimum deviation angle and minimum losses differ for each profile and correspond to different exit Mach numbers: for the P2 profile, this regime is close to $M_2 = 0.8$, while for the other profiles it lies closer to $M_2 = 0.9 - 0.92$.

It is also worth noting that for regimes close to the minimum deviation angle ($M_2 = 0.917$), choking does not occur (Fig. 15). This further confirms that M_2 is not a direct indicator of the minimum deviation angle.

From the standpoint of the influence of β_g on the overall (pitch-averaged) deviation angle, the results are consistent with literature data showing deviations of approximately 1.0° for $\beta_g = 20^\circ$. The figures indicate that within the subsonic range $M_2 = 0.5 - 1.0$, which is typical for steam and gas turbine cascades, the deviation angle varies from -0.3° to $+1.2^\circ$. Despite this seemingly small variation, it corresponds to a flow rate prediction error of up to 5 % at $\beta_2 = 68^\circ$.

CFD results also shows that the deviation angle is influenced both by the profile geometry and by the flow structure within the inter-blade channel.

On one hand, geometric parameters (at a fixed β_g) clearly determine the trends of the exit angle. For example, profiles with smaller metal outlet angles (i.e., tangential direction measurement system) tend to exhibit smaller deviation angles and, consequently, smaller exit flow angles.

On the other hand, the influence of the boundary layer formed on both the pressure and suction sides of the blade must be considered. The growth of the boundary layer on the suction side, in particular, leads to a shift of the main flow core toward larger exit angles (tan.measure system).

In addition, a complex interaction is observed between the main flow core and the trailing-edge wake, which is formed both by the trailing edge itself and by the accumulated boundary layer as the flow leaves the blade surface.

The results of the literature review and the present study demonstrate that predicting the deviation angle is a complex, multifactorial problem requiring consideration of a wide range of geometric and operating regimes.

Further research aimed to improve prediction accuracy should account for the full spectrum of these influencing factors.

Table 3 – Geometrical parameters of studied profiles

Parameter	VKI (GO) [15]	H2 [18]	P2 [3], [18], [19]	P9025B
b_c , mm	58.4	58.4	58.4	58.4
β_g , ax.(tan.)* deg	67.92 (22.08)	67.92 (22.08)	67.92 (22.08)	67.92 (22.08)
β_{1m} , ax.(tan.) deg	26.20 (63.80)	17.80 (72.20)	58.40 (31.60)	9.20 (80.80)
β_{2m} , ax.(tan.) deg	64.70 (25.30)	67.92 (22.08)	68.30 (21.70)	67.72 (22.28)
γ , deg	33.2	37.2	12.0	47.4
r_{LE} , mm	2.89	3.5	1.58	0.51
r_{TE} , mm	1.44	0.17	0.47	0.45

* tan. – flow angle measured from the tangential direction; ax. – flow angles measured from the axial direction.

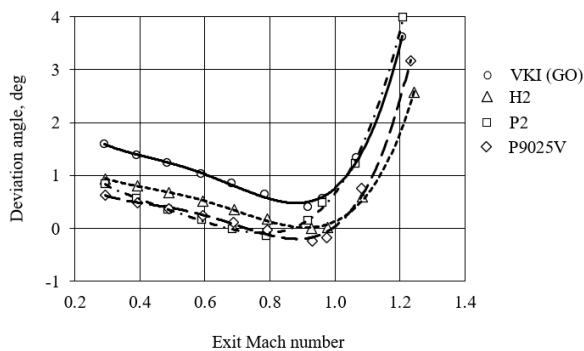


Fig. 13 – Deviation angle depending on the exit Mach number

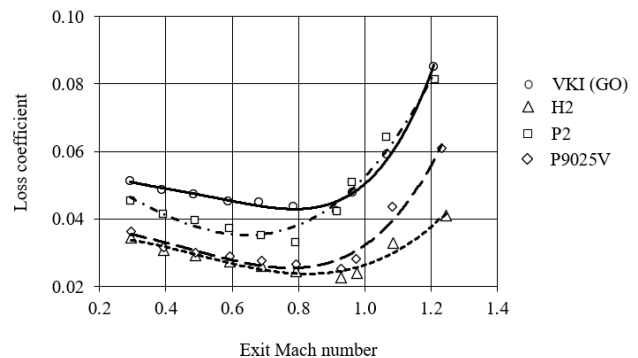


Fig. 14 – Loss coefficient depending on the exit Mach number

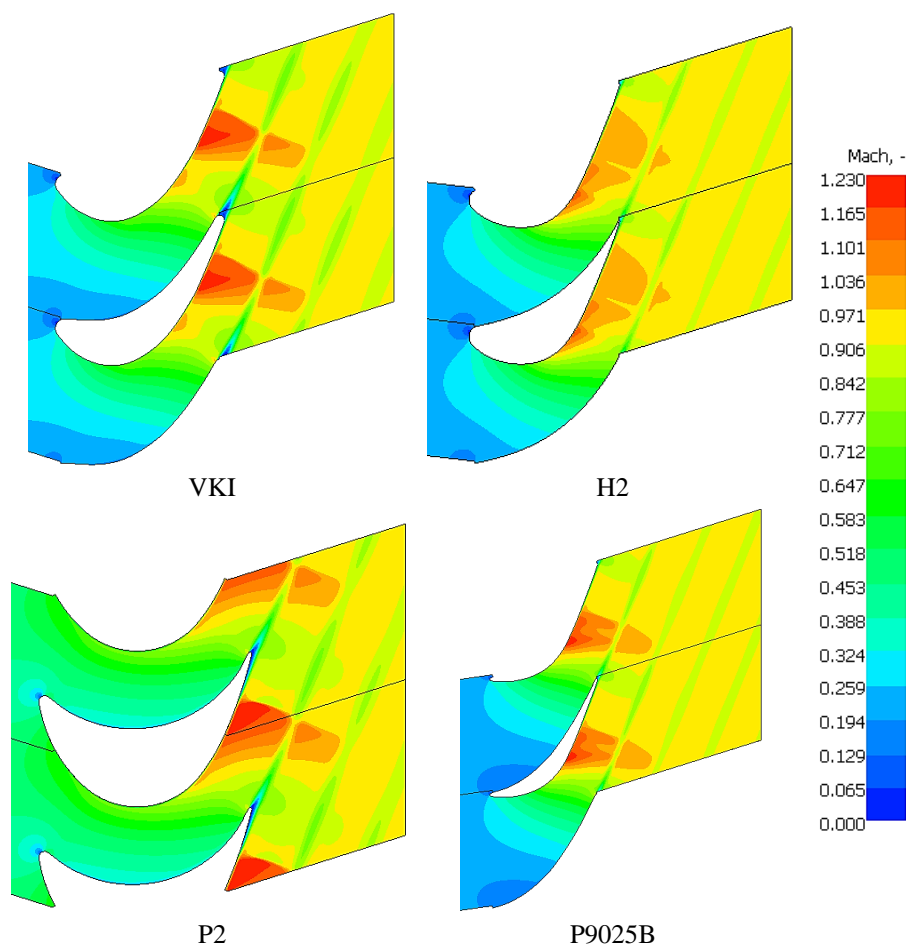


Fig. 15 – Mach number distribution in cascades ($M_2 = 0.917$)

Conclusions

The article presents a review of empirical correlations for predicting the flow exit angle from turbine cascades. The analysis of the considered methods showed that their accuracy varies depending on how geometric and operating regimes are considered.

A comparison was carried out between the results obtained using these methods and experimental data for the VKI cascade profile, both at the design point and off-design conditions.

The CFD analysis presented in the article demonstrated the closest agreement with the experimental data, indicating the feasibility of using CFD simulations to refine existing empirical correlations or to develop new ones. The results obtained using the AxCFD™ software show good agreement both in terms of the absolute values of the deviation angle and the trend of its variation under off-design conditions.

Additional CFD simulations of three planar turbine cascades with different profile geometries but the same gauging angle showed that the profile geometry has a noticeable impact on the flow exit angle, with the deviation angle variation from -0.3° to $+1.8^\circ$ for subsonic regimes.

It is shown that the minimum losses and the minimum deviation angle are interrelated, as evidenced both by literature data and by the performed CFD simulations. Therefore, to accurately account for the deviation angle and, consequently, the cascade flow coefficient, it is necessary to consider variations in boundary layer thickness within the cascade passage. This indicates the need for further detailed investigations in this area to better understand all factors affecting the deviation angle.

The results of the analysis of empirical correlations for the deviation angle, along with the CFD simulations performed, can be used to improve engineering calculation methods, thereby enhancing the accuracy of turbine stage design.

Data availability statement

The data supporting the findings of this research study consist of:

- 1) Published data from previously reported studies, which are properly cited in the article.
- 2) Original data generated by the authors through CFD simulations. The original datasets (including computed flow fields, Mach number distributions, exit flow angles, and loss coefficients) are available from the corresponding author upon reasonable request.

Author contributions: CRediT taxonomy

Babaiev A.: supervision, conceptualization, data analysis, writing original draft.

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Conflicts of interest

The authors declare no conflict of interest. SoftInWay, Inc. provided access to the software suite SoftInWay, Inc. AxSTREAM® for research purposes but had no role in the study design, data analysis, interpretation of results, or manuscript preparation.

Use of Artificial Intelligence

During the preparation of this manuscript, the authors used artificial intelligence tools (in particular GPT-5.3-mini) solely for retrieving publicly available information, language editing, and improving the clarity and style of the text. All scientific results, analysis, and conclusions were developed independently by the authors.

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